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Toward a Unification of "Everything" with Gravity

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Combining the ideas of gauge interactions with a global supersymmetry, we build a unified model in six dimensions step by step, starting with a single generation of leptons and ending with three generations of leptons and colored quarks forming a supermultiplet characterized by a most general extension $N = 8$. The puzzle of supersymmetric partners, such as the gravitino, photino, s-leptons, and s-quarks, is seen in a new light. The supersymmetry is only a global one, whereas local supersymmetry and supergravity are replaced by the theory of gauge interactions and by the usual general relativity of Einstein.

Let us begin with a simple model of one generation of leptons interacting electroweakly and gravitionally. Consider a set of fields with the same number of fermionic and bosonic degrees of freedom consisting of one tensor field, two Rarita–Schwinger fields, four vector fields, six Weyl fields, and six scalars. All these fields are at first massless two-component fields except for one-component scalars. This set may be split either into ($N = 1$)-supermultiplets or into extended ($N = 2$)-supermultiplets (the rows denote fields with spins 2, 3/2, 1/2, and 0):

Table 1

$$\begin{bmatrix} 1 \\ 2 \\ 4 \\ 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ - \\ - \\ - \end{bmatrix} + \begin{bmatrix} - \\ 1 \\ 1 \\ - \\ - \end{bmatrix} + 3 \times \begin{bmatrix} - \\ - \\ 1 \\ 1 \\ - \end{bmatrix} + 3 \times \begin{bmatrix} - \\ - \\ - \\ 1 \\ 2 \end{bmatrix}$$

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Table II

$$\begin{bmatrix} 1 \\ 2 \\ 4 \\ 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ — \\ — \end{bmatrix} + 3 \times \begin{bmatrix} — \\ — \\ 1 \\ 2 \\ 2 \end{bmatrix}$$

The first column on the right of Table I is interpretable as the graviton and gravitino. The second represents the photon and photino, whose spin is unexpectedly not smaller, but higher by 1/2 than unity. The triplet appearing in the third column denotes three vector fields $W^{\pm}$ and Z^0 mediating weak interactions and is combined with a triplet of their supersymmetric partners with spins 1/2 to be called W-ino and Z-ino. A triplet of Weyl fields appearing in the last but one column may be interpreted also as a triplet of weakly interacting leptons, possibly e_R, e_L, and ν_L^e. Two of them, namely e_R, e_L, may be fused into a 4-component Dirac field, but the third Weyl field has no partner with an opposite helicity and consequently exhibits a chiral character of the weak interactions.

The next problem is that of spontaneous symmetry breaking. Three of the six scalars appearing in Tables I and II should be swallowed up by the triplet of vector fields appearing in the third row of Table I endowing them with big masses in agreement with experimental evidence. It confirms our previous guess that these fields are $W^{\pm}$ and Z^0. Similarly, two of the Weyl fields have also to be swallowed up by the two spin-3/2 fields, endowing them with very big masses, which explains why corresponding particles have not been observed yet. The following table shows the results of spontaneous symmetry breakdown.

Table III

$$\begin{bmatrix} 1 \\ 2 \\ 4 \\ 6 \\ 6 \end{bmatrix} \to \begin{bmatrix} 1 \\ 2' \\ 1+3' \\ 1+3 \\ 1+2 \end{bmatrix}$$

where "prime" denotes "heavy." It is seen that besides a triplet of leptons, electron, and left-handed neutrino, one more Weyl field is left and may be regarded as a right-handed neutrino. It is excluded from weak interactions, but it is involved in a supersymmetric interaction within a local superdoublet with two scalars. According to the presence of a triplet and a singlet of vector fields, the symmetry of gauge interactions is $G = U(1) \times SU(2)$. The above model of lepton generation appears much simpler and intelligible if looked upon from a six-dimensional viewpoint. Let us assume that space-time is six-dimensional with topology of $M_4 \times S_2$ or $AdS \times S_2$ where S_2 is a two-dimensional spherical surface. Its radius is assumed to be extremely small. The metric field is

$$(g_{MN}) = \left(\begin{array}{c|c} g_{\mu\nu} & g_{\mu\nu} \\ \hline g_{\xi\nu} & g_{\xi\eta} \end{array}\right) \tag{1}$$

with $M, N = 0, 1, \ldots, 5$; $\mu, \nu = 0, \ldots, 3$; and $\xi, \eta = 4, 5$. The mixed metric tensor components $g_{\mu\xi}$ appear as if components of two four-vectors assume the form (Witten, 1981).

$$g_\mu \xi = \sum_{a=1}^{3} A^a_\mu K^a_\xi \tag{2}$$

where K^a_ξ are the Killing vectors of a sphere. The fields A^a_μ are to be identified just with $W^\pm_\mu$ and Z^0_μ exhibiting the symmetry of a sphere, being a fundamental representation of $SU(2)$. From the point of view of a Minkowskian observer the components $g_{\xi\eta}$ look like scalars.

Three four-vectors A^a_μ have been incorporated intrinsically into the six-dimensional metric field g_{MN} in agreement with Kaluza's original assumption; however, the electromagnetic field is not interpretable as a constituent of a generalized metric, but denotes the first four components of a six-vector:

$$V_M = \{V_\mu, V_\xi\} \tag{3}$$

It follows that the common view that all vector fields have a metrical origin may be regarded as a prejudice. The extra components of the six-vector V_ξ form something like a "tail" and look like scalars for macroscopic observers. They may be identified with two apparent scalars appearing in the last row of Table III so that the number of genuine scalars in our scheme reduces to a singlet, a Goldstone scalar.

Inasmuch as the number of independent components of the generalized metric field is 11 (two $g_{\mu\nu}$, six A^a_μ, and three $g_{\xi\eta}$), while that of massless Rarita–Schwinger field components in $D = 6$ is 12, whereas the numbers of components of massless vector fields as well as of Weyl spinors in $D = 6$ is four, Table I may be rewritten and simplified enormously, as follows:

Table IV

$$\begin{bmatrix} 1 \\ 2 \\ 4 \\ 6 \\ 6 \end{bmatrix}_{D=4} \rightarrow \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}_{D=6}$$

which justifies *ex post* our initial choice of the multiplet appearing on the left-hand sides of Tables I–IV.

Three Generations of Leptons

In order to perform a transition to a triplet of leptonic generations we proceed as follows: Assume a (reducible) supermultiplet involving 1 tensor field, 4 Rarita–Schwinger spinors, 12 vector fields, 24 Weyl spinors, and 30 scalars. This supermultiplet consists of 56 fermionic and 56 bosonic degrees of freedom and splits into irreducible supermultipets characterized by the following indices of extension: once $N = 4$, six times $N = 2$, and eight times $N = 1$, as seen from the following table:

Table V

$$\begin{bmatrix} 1 \\ 4 \\ 12 \\ 24 \\ 30 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 6 \\ 4 \\ 2 \end{bmatrix} + 6 \times \begin{bmatrix} - \\ - \\ 1 \\ 2 \\ 2 \end{bmatrix} + 8 \times \begin{bmatrix} - \\ - \\ - \\ 1 \\ 2 \end{bmatrix}$$

Applying the Higgs mechanism for spontaneous symmetry breaking, 11 of 12 vector fields acquire considerable masses by swallowing up 11 scalar fields. Similarly, four Rarita–Schwinger fields become very heavy, too, by swallowing up four of the Weyl spinors, as is seen from the following table:

Table VI

$$\begin{bmatrix} 1 \\ 4 \\ 12 \\ 24 \\ 30 \end{bmatrix} \xrightarrow[ssb]{} \begin{bmatrix} 1 \\ 4' \\ 1 + 3' + 8' \\ 12 + 8 \\ 18 + 1 \end{bmatrix}_{D=4} = \begin{bmatrix} 1 \\ 2' \\ 1 + 8' \\ 6 \\ 1 \end{bmatrix}_{D=6}$$

According to the appearance of a set of $1 + 3' + 8'$ vector fields in the third row of the middle column of Table VI, it is seen that the gauge group is

$$G_l = U(1) \times SU(2) \times SU(3) \tag{4}$$

due to the fact that the octet of heavy vector fields means a fundamental representation of the $SU(3)$ group. The particles forming this octet may be called para-gluons. They must possess a considerable mass value in order to prevent a quick decay of higher into lower generations.

The adequacy of the $SU(3)$ symmetry group is confined also by a discussion of the set of fields appearing in the fourth column of Table VI. Their number, 24, splits naturally into $12 + 8 + 4$, whence four have to be swallowed up by the Rarita–Schwinger fields, a further eight are also related to Rarita–Schwinger fields inasmuch as they form their "tails" if going over to a six-dimensional description, so that finally we are left with only 12 two-component (or six four-component) Weyl spinors in four (or six) dimensions. These numbers are factorizable by the factor 3, namely $12 = 2 \times 2 \times 3$, which means that the corresponding spinor fields form triplets. Thus the number 12 of Weyl spinors denotes nothing else but three generations of leptons. They include also right-handed neutrinos, although the latter do not participate in weak interactions.

The 18 scalars appearing in the last row of the middle column of Table VI mean "tails," i.e., additional components of the $8 + 1$ six-vectors, so that finally we are left with only one single genuine scalar, a Goldstone boson, if regarding and interpreting the multiplet from a 6-dimensional viewpoint.

Three Generations of Colored Quarks

Let us consider now a supermultiplet consisting of 96 bosonic and 96 fermionic degrees of freedom, namely 1 tensor, 6 Rarita–Schwinger spinors (spin 3/2), 20 vectors, 42 Weyl spinors, and 54 scalars. It forms a (reducible) supermultiplet splitting into the following irreducible constituents:

Table VII

$$\begin{bmatrix} 1 \\ 6 \\ 20 \\ 43 \\ 54 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 16 \\ 26 \\ 30 \end{bmatrix} + 4 \times \begin{bmatrix} - \\ - \\ 1 \\ 4 \\ 6 \end{bmatrix}$$

The irreducible constituents are: a single $(N = 6)$-extension and a quartet of $(N = 4)$-extensions with the highest spin values 2 and 1, respectively.

The number of vector fields splits as follows: $20 = 1 + 3 + 2 \times 8$, and is compatible with a symmetry group of gauge interactions

$$G_{qg} = U(1) \times SU(2)_L \times SU(3)_g \times SU(3)_c \tag{5}$$

where one of the two octets is related to the symmetry group of three generations, the other with the group of color. In order to prevent a quick decay of higher into lower generations the para-gluons must be massive, which may be achieved by a (generalized) Higgs mechanism. Three vector fields representing the group $SU(2)_L$ together with eight representing the group $SU(3)_g$ have to swallow up 11 scalars, while the six spin-3/2 fermions must swallow up six Weyl spinors, endowing the respective particles with high masses:

Table VIII

$$\begin{bmatrix} 1 \\ 6 \\ 20 \\ 42 \\ 54 \end{bmatrix} \xrightarrow[ssb]{} \begin{bmatrix} 1 \\ 6' \\ 1+3'+8+\underline{8}' \\ 36 \\ 34+9 \end{bmatrix}_{D=4} = \begin{bmatrix} 1 \\ 3' \\ 1+8+8' \\ 12 \\ 8+1 \end{bmatrix}_{D=6}$$

From the fourth row of the middle column it is seen that the number of Weyl fields is 36, interpretable as three generations of colored quarks according to the splitting $36 = 2 \times 2 \times 3 \times 3$, where 2×2 denotes a doublet of helicities times a doublet of charm or flavor whereas 3×3 denotes a triplet of generations times a triplet of color. It should be stressed that the octet of lepton and quark generations must be different one from the other in order to prevent decay of quarks and hadrons into leptons. In order to account for the masses of leptons that $SU(3)$ symmetries of generations must be (slightly) broken, but the mechanisms of these breakdowns are not yet clear.

The Problem of an $N = 8$ Extension

It is suggestive to assume that the set of all fundamental fields and particle types existing in nature should form the most general, irreducible supermultiplet characterized by the index of extension $N = 8$. This supermultiplet includes 1 tensor, 8 Rarita–Schwinger fields, 28 vectors, 56 Weyl fields, and 70 scalars. In view of the splitting $28 = 1 + 3 + 8 \times 3$ it could be supposed that the symmetry group of gauge interactions is

$$G = U(1) \times SU(2) \times SU3 \times SU(3) \times SU(3) \tag{6}$$

of rank 8. However, it seems impossible to "put quarks and leptons into one basket." Inasmuch as strong gauge interactions are not universal and quarks interact with leptons only via universal gravitational and electroweak couplings, we assume that the Lagrangian splits into leptonic and quarkonic parts interacting only via subgroups (4) and (5) of the most general possible group (6) of gauge interactions of ranks 4 and 6, respectively. The generality of the scheme will be preserved only insofar as *all* fields appearing within the ($N=8$)-multiplet partake either in the leptonic or quarkonic parts of the Lagrangian

$$\mathscr{L} = \mathscr{L}_l + \mathscr{L}_q + \mathscr{L}_b \tag{7}$$

The lepton and quark parts $\mathscr{L}_l$ and $\mathscr{L}_q$ involve the interaction terms with bosons, whereas the bosonic part $\mathscr{L}_b$ denotes a sum of interaction-free bosonic fields. The gauge groups of interactions with leptons and quarks in $\mathscr{L}_1$ and $\mathscr{L}_q$ are the groups (4) and (5), respectively. If all interactions are of a gauge type and usual gravitational ones as well as of Yukawa–Higgs type, then writing down a Lagrangian (7) is rather a matter of standard techniques.

Assume that the triplet $W^{\pm}Z^0$ as well as both para-gluonic octets become massive as visualized by the following table:

Table IX

$$\begin{bmatrix} 1 \\ 8 \\ 28 \\ 56 \\ 70 \end{bmatrix} = \begin{bmatrix} 1 \\ 8' \\ 1+3'+8+8'+8' \\ 48 \\ 1+50 \end{bmatrix}$$

The large masses of the two octets of para-gluons prevent a possibility of a quick decay of higher into lower generations. The appearance of two different octets of para-gluons for leptons and quarks assures a lack of decay of hadrons into leptons.

The reduction of the number of Weyl spinors from 56 to 48 is just sufficient and necessary to interpret the remaining 48 as three generations of leptons and quarks. To see this, let us perform the following splitting:

$$48 = 12 + 36 = 2 \times 2 \times 3 + 2 \times 2 \times 2 \times 3 \times 3$$

where 12 denotes the number of leptons and 36 that of quarks. As before 2×2 means two helicities times a doublet of charm (or flavor). One of the triplets accounts for the three generations of leptons or quarks, while the second triplet accounts for the color of quarks. Nevertheless, the multiplet $N = 8$ is not simply a sum of formerly discussed lepton and quark multiplets, because of different roles of universal electron weakly-gravitational and specific $SU(3)$ interactions. Adding simply the schemes would mean doubling the gravitational field and the number of fields representing the $W^{\pm}$, Z^0 bosons. Instead, we may perform a decomposition according to the following table, where the first column to the right-hand side denotes the leptonic sector, the third describes the quarkonic sector, while the middle column denotes their common part, i.e., the terms joining leptonic with quarkonic worlds via universal gravitational, electroweak, and Yukawa-like interactions:

Table X

$$
\begin{bmatrix} 1 \\ 8 \\ 28 \\ 56 \\ 70 \end{bmatrix} = \begin{bmatrix} - \\ 2 \\ 8 \\ 12+2 \\ 16 \end{bmatrix} + \begin{bmatrix} 1 \\ - \\ 4 \\ - \\ 22 \end{bmatrix} + \begin{bmatrix} - \\ 6 \\ 16 \\ 36+6 \\ 32 \end{bmatrix}
$$

The additional numbers 2 and 6 of Weyl spinors from the fourth rows of Table X are to be swallowed up by the corresponding spin-3/2 fields, endowing them with masses by means of a mechanism of spontaneous symmetry breaking. The numbers of scalars are explicable if we reinterpret the supermultiplet from a six-dimensional viewpoint. The 16 and 32 scalars appearing in the first and third columns are the "tails" of the corresponding six-vectors, whereas the number of $22 = 1 + 3 + 2 + 16$ scalars from the middle row denotes, respectively, a Goldstone boson, a triplet of metric tensor components $g_{\xi\eta}$, the "tail" of the six-vector of electromagnetic potentials, and a set of further 16 apparent scalars which will be swallowed up by the two octets of para-gluons endowing them with masses.

If reinterpreted from a four-dimensional to a six-dimensional viewpoint the $(N = 8)$-dimensional extended supermultiplet assumes the following form:

Table XI

$$
\begin{bmatrix} 1 \\ 8 \\ 28 \\ 56 \\ 70 \end{bmatrix}_{D=4} \rightarrow \begin{bmatrix} 1 \\ 1+3 \\ 25 \\ 5+15 \\ 1+16 \end{bmatrix}_{D=6}
$$

(still prior to the spontaneous symmetry breakdown).

Let us notice the following remarkable circumstances: the numbers of Weyl spinors viewed from a six-dimensional perspective are odd, namely 5 and 15 in the fourth row of table (18) if decomposed into leptonic and quarkonic sectors, i.e., into singlets and triplets. This explains why parity conservation must be violated by weak interactions. Weyl spinors cannot be fused into Dirac spinors in $D = 6$.

CONCLUDING REMARKS

Our tables may be regarded as analogs of the Mendelev table of chemical elements, but this time applied to elementary particles. Similarly, as the original table of Mendelev exhibited some signs of periodicity (and therefore was called the "periodic table of elements"), also here we notice signs of periodicity, since the rows denote bosonic and fermionic fields alternately. Moreover, as in the case of the Mendelev table, there appear also here empty places to be filled up by some expected but not yet discovered elements (particle types).

Table XI reveals also the possibility of another gauge symmetry, namely $U(1) \times SU(2) \times SU(5)$. This possibility should be worked out also. The fact that our scheme fits so well and smoothly into a six-dimensional framework shows decisively the inadequacy of the concept of superstrings, the latter requiring at least a 10-dimensional space.

In spite of the fact that now the problem of writing down explicitly a Lagrangian for a unified theory is only a matter of standard techniques, the above assumptions cannot be regarded yet as a full unification because they do not predict all possible relations between coupling constants, provided they are indeed constants if viewed from the point of view of a cosmological time scale.

A possible objection that the above models may turn out to be mathematically inconsistent (because of nonrenormalizable coupling, etc.) is to be refuted, since obviously it does not apply only to such or similar endeavors of unification, but equally well to any contemporary quantum formalism of gravitation. Probably such objections cannot be avoided unless a profound modification of the concept of general relativity is achieved (Rayski, 1984, 1992).

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